

of their rectangular shape. This book gives a systematic treatment of the Walsh–Fourier series and transforms, and some of their applications. Chapters 1–5 and Chaps. 7–9 deal with the theory of Walsh–Fourier series (basic properties, uniqueness,  $(C, 1)$  sums, uses of the Hardy–Littlewood maximal operator, convergence in  $L^p$ , etc.). Chapter 10 covers approximations by Walsh systems. Multiplicative transforms and their applications may be found in Chaps. 6, 11, and 12. The appendices are quite handy as they provide much useful background information. This makes the book accessible to a wide audience, which could certainly include beginning graduate students.

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A. ISERLES AND S. P. NØRSETT, *Order Stars*, Chapman & Hall, 1991, 248 pp.

Numerical algorithms for solving differential equations (ordinary and partial) require at least two important properties: they have to converge to the actual solution with a certain order and they have to be stable in the sense that a local change should not affect the solution globally. Quite often the numerical method is completely determined by a rational function  $R$ . The order of the numerical method can then be obtained by the degree of interpolation of  $R$  to a certain function  $f$  and stability is equivalent to a condition on the ratio  $R/f$  in a portion of the complex plane. Order stars give a technique for examining order and stability. Let  $f$  be a meromorphic function (a finite number of essential singularities are also allowed) and let  $R$  be a rational approximation to  $f$ . If  $\mathbf{C}^* = \mathbf{C} \cup \{\infty\}$ , then the order star of  $\{f, R\}$  is the triplet  $\{\mathcal{A}_+, \mathcal{A}_0, \mathcal{A}_-\}$  where  $\mathcal{A}_+ = \{z \in \mathbf{C}^*: |R(z)/f(z)| > 1\}$ ,  $\mathcal{A}_0 = \{z \in \mathbf{C}^*: |R(z)/f(z)| = 1\}$ , and  $\mathcal{A}_- = \{z \in \mathbf{C}^*: |R(z)/f(z)| < 1\}$ . Information about interpolation points is in  $\mathcal{A}_0$ ; information about zeros of  $R$  and poles of  $f$  is in  $\mathcal{A}_-$ ; poles of  $R$  and zeros of  $f$  are to be found in  $\mathcal{A}_+$ . Particular attention is paid to rational approximants for the exponential function because Runge–Kutta methods and Obrechhoff methods for solving  $y' = f(t, y)$  for  $t \in [a, b]$  and  $y(a) = y_0$  naturally lead to such approximants. Padé approximants are very natural here because they maximize the order of approximation at one point. However, quite often maximal order does not imply stability, so there has to be an interaction between trying to achieve high order while maintaining stability. One way to achieve this is to put restrictions on the zeros or poles of the approximants, which leads to  $z$ -restricted Padé approximants (restrictions on the zeros) and  $p$ -restricted Padé approximants (restrictions on the poles). Order stars are also used to analyze partial differential equations, especially the advection equation  $\partial u/\partial t = \partial u/\partial x$  and the diffusion equation  $\partial u/\partial t = c \partial^2 u/\partial x^2$  ( $c > 0$ ). Order stars can also give interesting results in the theory of rational approximation. It is well known that the Padé tableaux for Padé approximants to a function  $f$  consists of square blocks, such that every block contains the same approximant for  $f$ . An important number is the maximal block size  $\beta(f)$  in the Padé tableau for  $f$ . The authors use order stars to find some useful upper bounds on  $\beta(f)$ . Order stars are also used to give some results on approximants that map an open set into the open unit disc (contractive approximation). As an example the authors give a brief outline of the Pick–Nevanlinna interpolation problem. The book is nicely illustrated with many pictures of order stars, which in the spirit of the topic is very helpful and an absolute necessity. Indeed, it is by looking at order stars that many properties of the corresponding numerical method or approximation problem are revealed. Also very helpful is a description of 13 open problems. A nice piece of mathematics on the interaction between theory and practice.

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